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# Experimental study on bubble velocity, void fraction and pressure drop for gas–liquid two-phase flow in a circular microchannel $\stackrel{\star}{\sim}$

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# ABSTRACT

An adiabatic experiment was conducted to investigate the effects of liquid properties on the characteristics of two-phase flows in a horizontal circular microchannel. Distilled water and aqueous solutions of ethanol were used as the test liquids. The ethanol concentration was varied to change the surface tension and the viscosity. One of the four liquids together with nitrogen gas was injected through a T-junction mixer to the test microchannel. Two mixers with different inner diameters of  $D_M = 250 \,\mu\text{m}$  and 500  $\mu\text{m}$  were used at a fixed microchannel diameter of  $D = 250 \,\mu\text{m}$  to study flow contraction effects at the channel inlet. Bubble velocity data correlated with the drift flux model showed that the distribution parameter,  $C_0$ , increased with increasing of liquid viscosity and/or decreasing of surface tension, and  $C_0$ for flows with the contraction was higher. The pressure drop data correlated with the Lockhart–Martinelli method showed that the two-phase friction multiplier,  $\phi_L^2$ , for flows with the contraction was lower. From data analysis, new correlations of  $C_0$  and  $\phi_L^2$  were developed with some dimensionless numbers. On void fraction prediction, two-fluid model code could predict well the data when an appropriate correlation of interfacial friction force was used.

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### 1. Introduction

Vigorous studies are underway to understand the movement of a micro-scale fluid. Especially, the understanding of gas–liquid two-phase flow characteristics in a microchannel is essential for developing and designing micro-devices such as microreactor (Jähnisch et al., 2000), mobile type fuel cell (Yen et al., 2003) and micro-heat exchangers (Qu and Mudawar, 2003), etc. In the flow characteristics, Serizawa et al. (2002), Kawahara et al. (2002, 2003, 2004, 2005a,b, 2006), Chung et al. (2004), Chung and Kawaji (2004) and Kawaji et al. (2006) reported unique differences between the microchannel and the conventional sized channel.

Serizawa et al. (2002) conducted air–water two-phase flow experiments in 25  $\mu$ m and 100  $\mu$ m horizontal microchannels, and observed five flow patterns: dispersed bubbly, gas slug, liquid-ring, liquid lump and liquid droplet flows. The existence of the liquid-ring flow is a difference between the microchannel and the conventional sized channel.

Kawahara et al. (2002) and Chung and Kawaji (2004) studied two-phase flows of nitrogen gas and water through horizontal microchannels of  $50-250 \,\mu$ m diameter, and reported significant

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differences in the flow pattern maps and void fraction data from the conventional sized channels. In addition, they reported the existence of flow patterns unique to microchannels, e.g., liquidring flow and serpentine-like gas core flow, and void fraction data showed a strong derivation from Armand (1946)-type correlation normally applicable to the conventional sized channel and minichannel (Ali et al., 1993). Such a trend of void fraction data is in contrast with Serizawa et al. (2002) data for a 25  $\mu$ m horizontal microchannel, which were well represented by the Armand type correlation.

Following to the above studies, Kawahara et al. (2006) and Kawaji et al. (2006) conducted adiabatic experiments to clarify the effects of gas and liquid injection methods and inlet geometry on water/nitrogen gas two-phase flow in microchannels by changing a combination of the mixer and the microchannels. Horizontal circular microchannels of 100, 176 and 251  $\mu$ m I.D. were connected in turn to one of the two mixers of 250 and 500  $\mu$ m I.D. Water and nitrogen gas were used as the working fluids. Two types of flow configuration were mainly observed. They were called "quasihomogeneous flow" and "quasi-separated flow". Interestingly, the void fraction in the quasi-homogeneous flow was higher than that in the quasi-separated one at the same gas and liquid flow rates condition.

Some papers (e.g., Chung and Kawaji, 2004) noted that with decreasing channel size, the Bond number, Reynolds number, the capillary number all decrease. Thus, compared to two-phase flows

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# Nomenclature

| а                | constant in Eq. (7)                        | We         | Weber number                  |
|------------------|--|------------|-------------------------------|
| a <sub>INT</sub> | interfacial area concentration             | X          | Lockhart-Martinelli parameter |
| b                | constant in Eq. (14)                       | Ζ          | axial distance                |
| Во               | Bond number                                |            |                               |
| С                | constant in Eq. (11)                       | Greek sy   | rmbols                        |
| $C_0$            | distribution parameter                     | α          | void fraction                 |
| С                | Armand constant in Eq. (9)                 | β          | homogeneous void fraction     |
| Са               | Capillary number                           | $\phi_I^2$ | two-phase friction multiplier |
| $C_D$            | drag coefficient                           | μ          | dynamic viscosity             |
| Ď                | channel diameter                           | ρ          | density                       |
| $D_H$            | hydraulic diameter                         | $\sigma$   | surface tension               |
| $D_M$            | mixer diameter                             |            |                               |
| Ε                | error                                      | Subscrip   | ts                            |
| $F_I$            | interfacial friction force per unit volume | Cal.       | calculation                   |
| $F_W$            | wall friction force per unit volume        | Exp.       | experiment                    |
| g                | gravitational acceleration                 | G          | gas phase                     |
| ī                | volumetric flux                            | L          | liquid phase                  |
| $\tilde{L}_G$    | bubble length                              | m          | mean                          |
| Р                | pressure                                   | rms        | root-mean-square              |
| Re               | Reynolds number                            | TP         | two-phase                     |
| и                | mean velocity                              |            | I                             |
| U <sub>R</sub>   | relative velocity                          |            |                               |
| Vci              | drift velocity                             |            |                               |
| · •j             |  |            |                               |

in conventional sized channel, the flow in microchannel seems to be more influenced by surface tension effect and viscous force rather than gravity force and inertia. In order to develop devices utilizing such a microchannel, therefore, it is essential to know the effect of surface tension and/or viscosity on two-phase flow pattern, void fraction and pressure drop etc. However, the effects have not yet been revealed because most of the two-phase flow experiments in microchannels have been performed using water alone as the working liquid.

In this connection, the purpose of this study is to know experimentally the effects of liquid physical properties on the two-phase flow characteristics, such as the bubble velocity, the bubble length, the void fraction and the pressure drop. An adiabatic experiment was conducted to obtain such characteristic data for gas-liquid two-phase flows in a 250 µm circular microchannel. In order to study effects of surface tension and viscosity, aqueous solutions of ethanol having different mass concentrations of 0, 4.8, 49 and 100 wt% were used as the working liquid, while nitrogen gas as the working gas. In addition, to know the effects of flow contraction at the channel inlet, two mixers of different inner diameters of  $D_{\rm M}$  = 250 µm and 500 µm were used at a fixed microchannel diameter of  $D = 250 \,\mu\text{m}$ . In the analysis, the void fraction and pressure drop correlations reported so far are tested against the present data. Furthermore, on the void fraction, calculated value by a onedimensional two-fluid model code is also tested. Results of the experiment and the analysis are presented in this paper.

# 2. Experiments

#### 2.1. Test apparatus

Fig. 1 is the present test apparatus, being the same as that used in our previous study (Kawahara et al., 2006; Kawaji et al., 2006). As the test fluids, distilled water and aqueous solution of ethanol were used for the liquid phase, while nitrogen for the gas phase. The liquid was introduced to horizontal, circular microchannel by a pneumatic-type pump. The pump consisted of a pressure vessel containing one of the test liquids and a gas cylinder of dry nitrogen



Fig. 1. Test apparatus.

for pushing the liquid surface in the vessel. This pump gave a stable and pulsation-free liquid flow. All tubing and fittings were made of stainless steel or brass to avoid any volumetric expansion in the flow loop and fluids leakage by a high pressure. A gas and liquid mixture made at a mixer flowed through the circular microchannel test section and discharged to the atmosphere.

The liquid flow rate was determined by weighing the liquid discharged in a small container over a sufficient period of time with an electronic balance ( $200 \pm 0.001$  g, SHIMADZU Co.). The gas flow rate was read from a calibrated mass flow meter (5 SCCM, Type HM5111B, Tokyo Keiso Co., Ltd.).

The transparent test section made of a 250 µm I.D. fused silica (Polymicro Technologies Inc.) enabled us to observe flow with a high speed video camera (Hi-Dcam PCI 8000S, NAC Image Technology). Background illumination was provided by a high-intensity



Fig. 2. Gas-liquid mixer and the microchannel.

luminescent lamp (LA-180Me, Hayashi Tokei Kogyo Co.) and gooseneck light guide placed behind the test section. To record enlarged images of the flow inside the test section, a macro zoom lens (Z16APO, Leica Microsystems) was coupled to the video camera as shown in Fig. 1. The total length of the microchannel was L = 99 mm, and a high L/D ratio (L/D = 396) allowed us to diminish the entrance and exit effects. The window for the flow observation was located at the mid point of the test section.

Fig. 2 shows a gas–liquid mixer made from a T-junction (Valco Instrument Co., Inc.) with an inner diameter of  $D_M$  = 250 or 500 µm. Since the T-junction was directly connected to the 250 µm I.D. test section, when 500 µm I.D. mixer was used, flow contraction occurred at the inlet of the test section. Liquid was injected into the main channel (line 1), while the gas into the branch (line 2).

# 2.2. Bubble velocity, bubble length and void fraction measurement

The high speed video camera was used to determine velocity and length of the bubbles in the test section. The images of flow with a resolution of  $160 \times 78$  pixels were recorded at 8000 frames per second and at a shutter speed of 1/160,000 s. The recorded images were transmitted to a computer for image processing by a commercial software.

The bubble velocity,  $u_G$ , can be determined as

 $u_G = \Delta Z f. \tag{1}$ 

Here, *f* is the frame rate of the video camera and  $\Delta Z$  the moving distance of the bubble nose during 1/f.

The bubble length,  $L_G$ , was determined by measuring the distance from the nose of the bubble to the tail if the bubble length was shorter than an image width (ca. 2.6 mm). If the bubble length was longer than the image width,  $L_G$  was determined by Agostini et al.'s method (2008):

$$L_G = u_G (F_{t,i} - F_{n,o}) / f.$$
(2)

Here,  $F_{t,i}$  is the frame number when the bubble tail appears on the video image,  $F_{n,o}$  the frame number when the bubble nose disappears on the video image.

The void fraction,  $\alpha$ , was determined by substituting  $u_G$  data into:

$$\alpha = j_G / u_G. \tag{3}$$

Here,  $j_G$  is the volumetric flux of the gas phase at the mid point of the test section.

#### 2.3. Determination of frictional pressure drop

The pressure drop measured in the present experiment,  $\Delta P_{\text{measured}}$ , is the sum of three components:

$$\Delta P_{\text{measured}} = \Delta P_{\text{friction}} + \Delta P_{\text{contraction}} + \Delta P_{\text{acceleration}}.$$
 (4)

Here,  $\Delta P_{\text{friction}}$  is the component of wall friction,  $\Delta P_{\text{contraction}}$  that of contraction at the microchannel inlet, and  $\Delta P_{\text{acceleration}}$  that of accel-

eration due to the expansion of the gas phase. In order to obtain the frictional component, the second and the third components on the right hand side of Eq. (4) must be subtracted from the pressure drop measured with a calibrated pressure transducer (FP101 Series, Yokogawa Co.). For that purpose,  $\Delta P_{\text{contraction}}$  and  $\Delta P_{\text{acceleration}}$  were estimated from a method described in Kawahara et al. (2002). The results showed that the contributions of the contraction and the acceleration components to the total pressure drop were, respectively 0.01% to 1.26% and 0.001% to 0.2%, depending on the flow conditions.

#### 2.4. Experimental conditions

Distilled water, aqueous solutions of ethanol with two different mass concentrations (49 wt% and 4.8 wt%) and pure ethanol were used as the working liquid. The density, dynamic viscosity and surface tension of each liquid are listed in Table 1. The deviation from the mean value depends on the change in liquid temperature in the experiments. A peculiar dependency of viscosity on the ethanol concentration is seen; it takes a maximum value at 49 wt%, and takes almost the same value at 100 wt% and 4.8 wt% though they are about three times different in surface tension.

The ranges of liquid and gas volumetric fluxes,  $j_L$  and  $j_G$ , are shown in Table 2. It should be noted that the volumetric flux of the gas phase was calculated using a gas density at the mid point of the test section which was evaluated by the system pressure and liquid temperature there.

The ranges of dimensionless numbers related to the present experiments are shown in Table 3. The significance of inertia force to viscous force is known from the liquid and the gas Reynolds numbers,  $Re_L(=\rho_L j_L D/\mu_L)$  and  $Re_C(=\rho_G j_G D/\mu_G)$ , while that of inertia force to surface tension force is indicated from the Weber numbers,  $We_L(=\rho_L j_L^2 D/\sigma)$  and  $We_G(=\rho_G j_G^2 D/\sigma)$ . That of gravity force to surface tension force, the Bond number,  $Bo(=(\rho_L - \rho_G)gD^2/\sigma)$ , was much smaller than unity in the present experiment. That of viscous force to surface tension force, the capillary number,  $Ca(=\mu_L j_L/\sigma)$ , was from 0.0024 to 0.851.

#### 3. Experimental results and discussions

#### 3.1. Flow pattern

Figs. 3 and 4 show typical flow without the flow contraction at the microchannel inlet, respectively for different working liquids.

| Table 1                        |  |
|--------------------------------|--|
| Properties of working liquids. |  |

| Working liquids | Density $\rho_L$ (kg/m <sup>3</sup> ) | Viscosity $\mu_L$<br>(mPa s) | Surface tension $\sigma$ (N/m) |
|-----------------|---------------------------------------|------------------------------|--------------------------------|
| Distilled water | 996.5 ± 1.7                           | 0.92 ± 0.1                   | 0.072 ± 0.001                  |
| Ethanol 4.8 wt% | 989.4 ± 1.6                           | $1.19 \pm 0.2$               | $0.060 \pm 0.001$              |
| Ethanol 49 wt%  | 910.9 ± 5.3                           | $2.43 \pm 0.5$               | $0.028 \pm 0.001$              |
| Ethanol 100 wt% | 785.7 ± 7.5                           | $1.16 \pm 0.2$               | $0.022 \pm 0.001$              |
|                 |                                       |                              |                                |

Table 2Ranges of volumetric fluxes for liquid and gas.

| Working liquids | <i>j</i> <sub>L</sub> (m/s) | $j_G(m/s)$ |
|-----------------|-----------------------------|------------|
| Distilled water | 0.22-1.43                   | 0.04-1.24  |
| Ethanol 4.8 wt% | 0.11-1.08                   | 0.07-0.96  |
| Ethanol 49 wt%  | 0.21-0.91                   | 0.04-1.77  |
| Ethanol 100 wt% | 0.22-1.52                   | 0.02-1.33  |

Ranges of dimensionless numbers.

| Working liquids | Re <sub>L</sub> | Re <sub>G</sub> | We <sub>L</sub> | We <sub>G</sub> | Во    | Са           |
|-----------------|-----------------|-----------------|-----------------|-----------------|-------|--------------|
| Distilled water | 52-386          | 0.6-24          | 0.16-7.12       | 0.00001-0.0070  | 0.008 | 0.0026-0.018 |
| Ethanol 4.8 wt% | 20-231          | 1.4-22          | 0.05-4.18       | 0.00003-0.0064  | 0.010 | 0.0024-0.021 |
| Ethanol 49 wt%  | 17-139          | 0.8-37          | 0.35-12.3       | 0.00002-0.043   | 0.020 | 0.019-0.851  |
| Ethanol 100 wt% | 33–287          | 0.5–26          | 0.41-20.9       | 0.00001-0.029   | 0.022 | 0.012-0.56   |



**Fig. 3.** Quasi-homogeneous flow observed for flows without flow contraction at  $j_L = 0.4$  m/s and  $j_G = 0.1$  m/s.

 $\leftarrow$  Flow direction



**Fig. 4.** Quasi-separated flow observed for flows without flow contraction at  $j_L = 0.4$  m/s and  $j_G = 1.0$  m/s.



**Fig. 5.** Effect of the flow contraction at  $j_L = 0.8$  m/s and  $j_G = 0.4$  m/s for water case.

Two types of flow pattern were observed. The first one, which is called as a quasi-homogeneous flow (Kawahara et al., 2006; Kawaji et al., 2006), is featured by the presence of gas plugs shorter than the width of viewing window (ca. 2.6 mm) as shown in Fig. 3. The second one, called as a quasi-separated flow (Kawahara et al., 2006; Kawaji et al., 2006), is characterized by a longer gas bubble surrounded by a smooth or a wavy liquid film, as shown in Fig. 4. The quasi-homogeneous flow tends to occur at relatively high liquid flux, while the quasi-separated flow at relatively low liquid and/or high gas fluxes.

Fig. 5 shows the effects of the flow contraction on the flow pattern. You can see longer bubbles for flows with the contraction even at the same gas and liquid flow rates condition.

# 3.2. Bubble velocity

Fig. 6a–d show the bubble velocity data,  $u_G$ , plotted against the total volumetric flux,  $j(=j_G + j_L)$ , for flows without and with the

flow contraction. The open symbol represents data for the flows without the contraction, while solid symbol for the flows with the contraction. The solid and broken lines represent regression lines of the data for the respective cases based on the well-known drift flux model (Zuber and Findlay, 1968):

$$u_G = C_0 j + V_{Gj}. \tag{5}$$

Here,  $C_0$  is the distribution parameter and  $V_{Gj}$  the drift velocity. In the regression,  $V_{Gj}$  was taken as zero because flows in the present study were horizontal. Table 4 shows the  $C_0$  data. From Fig. 6 and Table 4, it is found that  $u_G$  and  $C_0$  increased with increasing of the liquid viscosity and/or decreasing of the surface tension. The reason of this is probably that liquid film thickness around the gas bubble decreases with the increasing of the liquid viscosity, and the bubble nose shape become sharpen with increasing of the viscosity and/or the deceasing of the surface tension, as seen in Fig. 4. As for the contraction effects,  $C_0$  were higher for the flows with the contraction. The reason is presumably that the contraction elongates the bubble in the central region of the channel, and the bubble flows faster.

For air–water two-phase flow in vertical circular pipes of 1– 5 mm I.D., Mishima and Hibiki (1996) obtained the following  $C_0$  correlation:

$$C_0 = 1.2 + 0.510 \exp(-0.691D). \tag{6}$$

The substitution of D = 0.25 mm into Eq. (6) yields  $C_0 = 1.62$ , being much higher than  $C_0 = 1.10-1.22$  for water case in the present data. Thus, there is a room of improvement in Eq. (6) for flows of different working fluids in smaller channels than 1 mm I.D.

In order to develop the  $C_0$  correlation applicable to the different liquids–gas flows, the present  $C_0$  data were tried to correlate with various dimensionless numbers, and finally the following equation was obtained:

$$C_0 = aBo^{0.19} Re_I^{-0.01} We_C^{0.01}.$$
(7)

In Eq. (7), the constant *a* depends on flows without and with the contraction, i.e., a = 3.0 for the flow without the contraction, a = 3.3 for the flow with the contraction. Fig. 7a and b shows comparison of  $u_G$  between experiment and calculation by Eqs. (5) and (7). The calculations agreed well with the data for all the test liquids within r.m.s errors of 11.4%, irrespective of the flows without and with the contraction.



Fig. 6. Bubble velocity for four-kinds of test liquids-nitrogen gas two-phase flows without and with contraction.

**Table 4**Distribution parameter data for the present channel.

|                     | Water | Ethanol<br>4.8 wt% | Ethanol<br>49 wt% | Ethanol<br>100 wt% |
|---------------------|-------|--------------------|-------------------|--------------------|
| Without contraction | 1.10  | 1.09               | 1.40              | 1.34               |
| With contraction    | 1.22  | 1.38               | 1.57              | 1.35               |

#### 3.3. Bubble length

Fig. 8 shows the bubble length,  $L_G$ , data plotted against the homogeneous void fraction,  $\beta(=j_G/(j_G + j_L))$ , for the respective working liquid flows without and with the flow contraction. Significant contraction effects on  $L_G$  can be seen, that is,  $L_G$  is longer for flows with the contraction even in the same homogeneous void fraction,  $\beta$ , in  $\beta < 0.6$ . For flows with the contraction,  $L_G$  shows big scatters, depending mainly on liquid viscosity difference. However, for flows without the contraction, the scatter becomes small, and  $L_G$  is less than the channel inner diameter in  $\beta < 0.2$ , and increases with  $\beta$ .

The elongation of bubbles for flows with the contraction can be explained qualitatively as follows. Larger gas bubbles would be produced in the T-junction of  $D_M$  = 500 µm by a step upstream of the microchannel test section. When these bubbles are periodically released over the step and flow into the microchannel with a smaller diameter (D = 250 µm), the bubble length must be increased by

four times. For example, if the length of the gas bubble in the T-junction were 500  $\mu m,$  it would become a 2 mm in the microchanel.

# 3.4. Void fraction

Fig. 9 presents the void fraction,  $\alpha$ , data plotted against the homogeneous void fraction,  $\beta$ , for different working liquid flows without the flow contraction. For comparison, a dot-dash curve calculated by Kawahara et al.'s correlation (2002), Eq. (8), for nitrogen gas/water flow in a 100  $\mu$ m circular channel, is drawn on the figure.

$$\alpha = \frac{0.03\beta^{0.5}}{1 - 0.97\beta^{0.5}}.$$
(8)

In addition, two lines corresponding to a homogeneous flow line ( $\alpha = \beta$ ) and Armand's correlation (1946):

$$\alpha = \frac{1}{C_A}\beta, \quad C_A = 1.2, \tag{9}$$

are, respectively shown by solid and dashed lines. Ali et al. (1993) recommended the use of an Armand type correlation ( $\alpha = 0.8\beta$ ) for narrow rectangular channels with  $D_H \sim 1$  mm.

The void fraction data for distilled water and 4.8 wt% ethanol solution distributed between the lines for homogeneous flow model and Armand correlation. The data for 49 wt% ethanol solution and pure ethanol is lower than data for the above two test liquids.



**Fig. 7.** Comparison of bubble velocity between experiment and calculation by Eqs. (5) and (8).



Fig. 8. Bubble length against homogeneous void fraction.

In addition, the data for 49 wt% ethanol solution, having the highest viscosity among the test liquids, tend to approach the curve by Eq. (8).

In order to know the effects of the flow contraction on the void fraction, Fig. 10 compares void fraction data for 49 wt% ethanol solution flows without and with the contraction. The data for flows with the contraction were lower than that without the contraction at the same homogeneous void fraction,  $\beta$ , and apt to approach the curve by Eq. (8). The reason is presumably that the contraction elongates the bubble in the central region of the channel as shown in Figs. 5 and 8, and makes the bubble velocity faster.



Fig. 9. Void fraction for four-kinds of test liquids-nitrogen gas two-phase flow without contraction – effects of liquid properties on void fraction.



**Fig. 10.** Void fraction for ethanol 49 wt% aqueous solution–nitrogen gas two-phase flow – effects of the contraction on void fraction.

#### 3.5. Two-phase frictional pressure drop

The frictional pressure drop data are commonly correlated with the following two-phase friction multiplier,  $\varphi_L^2$  (Lockhart and Martinelli, 1949):

$$\left(\frac{dP_f}{dZ}\right)_{TP} = \phi_L^2 \left(\frac{dP_f}{dZ}\right)_L,\tag{10}$$

where  $(dP_f/dZ)_L$  is the frictional pressure drop when the liquid in two-phases flows alone in the same channel. A widely used correlation for the friction multiplier is that proposed by Chisholm and Laird (1958),

$$\phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2},\tag{11}$$

where X is the Lockhart–Martinelli parameter given by

$$X^{2} = \frac{\left(dP_{f}/dZ\right)_{L}}{\left(dP_{f}/dZ\right)_{G}}.$$
(12)

In Eq. (12),  $(dP_f/dZ)_G$  is the frictional pressure drop when the gas flows alone in the same channel.

The coefficient, C, in Eq. (11) is the constant ranging from 5 to 20 in conventional sized channels, depending on whether flows of the liquid and the gas are laminar or turbulent. According to Chisholm and Laird's criteria, the value of C for the present flow conditions must be five, because both the liquid and the gas are

in laminar flow, i.e.,  $Re_L(=\rho_L j_L D/\mu_L) < 3860$  and  $Re_G(=\rho_G j_G D/\mu_G) < 2370$ .

Mishima and Hibiki (1996) proposed a correlation of the *C*-value for their data on air–water flow in circular and rectangular channels of  $D_H$  = 1–4 mm as well as the data reported by other researchers:

$$C = 21(1 - e^{-0.319D_H}), \tag{13}$$

where  $D_H$  is the hydraulic diameter of the channel.

Fig. 11 shows a comparison of the two-phase friction multiplier data with the predictions by Eq. (11) with *C* = 5 and *C* = 1.61 given by Eq. (13). Also shown in the same figure is the calculated curve by Kawahara et al. (2002) with *C* = 0.24 for nitrogen gas/deionised water two-phase flow in a 100 µm circular channel. The data are well correlated with the Lockhart–Martinelli parameter, and an appropriate *C*-value seems to depend on the flow contraction. For flows with the contraction, the data agree reasonably with the calculation with *C* = 1.61, irrespective of the working liquids. For the flows without the contraction, on the other side, the data distribute around the calculation with *C* = 5. The reason why the friction multiplier is higher for flows without the contraction is as follows: For the flows without the contraction, the void fraction is higher as mentioned before, thus, mean liquid velocity,  $u_L=j_L/(1 - \alpha)$ , is faster, and resulting the wall friction higher.



Fig. 11. Two-phase friction multiplier versus Lockhart-Martinelli parameter.

The liquid properties seem to affect the *C*-value. So, the *C* data were obtained from the present frictional pressure drop data. The resulting *C* data were tried to correlate with three dimensionless numbers, and finally Eq. (14) was obtained:

$$C = bBo^{0.04} Re_{L}^{0.25} We_{C}^{-0.12}.$$
(14)

In Eq. (14), the constant *b* depends on the flows without and with the contraction, i.e., b = 1.38 for the flow without the contraction, b = 0.55 for the flow with the contraction. Fig. 12a and b presents a comparison of two-phase frictional multiplier between the experiment and the calculation by Eqs. (11) and (14). The calculation agreed well with the data within 20% r.m.s. errors, irrespective of the test liquids.

# 4. Analysis of pressure drop and void fraction

# 4.1. Two-phase frictional pressure drop prediction

In this section, eleven correlations developed for both macro and mini/micro-channels are confirmed to validate their suitability to the present two-phase frictional pressure drop data. The correlations tested are: homogeneous flow model with six different viscosity models (McAdams, 1954; Owens, 1961; Cicchitti et al., 1960; Dukler et al., 1964; Beattie and Whalley, 1982; Lin et al., 1991), Lockhart and Martinelli (L–M) model with four different *C* models (Chisholm and Laird, 1958; Mishima and Hibiki, 1996; Lee and Lee, 2001; Qu and Mudawar, 2003), and separated flow model (Ali et al., 1993). The L–M model with a newly developed *C* correlation, Eq. (14), was also tested. Table 5 lists the test results. The mean error,  $E_m$ , and the RMS error,  $E_{rms}$ , are defined as

$$E_{m} = \left[\frac{1}{N}\sum_{i=1}^{N}\frac{(dP_{f}/dZ)_{TP-Cal,i} - (dP_{f}/dZ)_{TP-Exp,i}}{(dP_{f}/dZ)_{TP-Exp,i}}\right] \times 100,$$
(15)  
$$E_{rms} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{(dP_{f}/dZ)_{TP-Cal,i} - (dP_{f}/dZ)_{TP-Exp,i}}{(dP_{f}/dZ)_{TP-Exp,i}}\right)^{2}} \times 100.$$
(16)

For flows without the contraction, L–M model with Chisholm and Laird's *C* model (1958) and the developed *C* correlation, Eq. (14), and separated flow model (Ali et al., 1993) give the best results, irrespective of working liquids. For flows with the contraction, L–M model with Mishima and Hibiki's *C* model (1996) and Lee and Lee's *C* model (2001), and the developed *C* correlation, Eq. (14), and homogeneous flow model with Dukler et al.'s viscos-



Fig. 12. Comparison of two-phase friction multiplier between experiments and calculation by Eqs. (11) and (14).

#### Table 5

Mean and RMS errors of various correlations for predicting two-phase frictional pressure gradient.

|                               | Distilled water    |                      | Ethanol 4.8        | Ethanol 4.8 wt%      |                    | Ethanol 49 wt%             |                    | Ethanol 100 wt%      |  |
|-------------------------------|--------------------|----------------------|--------------------|----------------------|--------------------|----------------------------|--------------------|----------------------|--|
|                               | E <sub>m</sub> (%) | E <sub>rms</sub> (%) | E <sub>m</sub> (%) | E <sub>rms</sub> (%) | E <sub>m</sub> (%) | <i>E<sub>rms</sub></i> (%) | E <sub>m</sub> (%) | E <sub>rms</sub> (%) |  |
| (a) Without contraction       |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Homogeneous flow type         |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Owens                         | -7.9               | 36.1                 | -8.9               | 15.0                 | 31.7               | 57.6                       | 1.7                | 29.8                 |  |
| McAdams                       | -13.9              | 30.6                 | -14.4              | 17.0                 | 8.5                | 23.5                       | -5.0               | 23.1                 |  |
| Cicchitti                     | -8.1               | 36.0                 | -9.0               | 15.0                 | 31.4               | 57.1                       | 1.6                | 29.7                 |  |
| Dukler et al.                 | -47.9              | 49.7                 | -43.7              | 46.7                 | -30.0              | 32.4                       | -34.9              | 36.5                 |  |
| Beattie and Whalley           | -0.8               | 18.9                 | 3.2                | 6.6                  | 36.8               | 43.1                       | 16.0               | 28.6                 |  |
| Lin et al.                    | -8.5               | 35.2                 | -9.4               | 14.9                 | 28.7               | 51.9                       | 1.1                | 28.9                 |  |
| Lockhart–Martinelli type      |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Chisholm and Laird            | -18.0              | 22.9                 | -16.8              | 19.8                 | -1.8               | 9.4                        | -4.9               | 14.1                 |  |
| Mishima and Hibiki            | -38.3              | 40.2                 | -35.0              | 37.9                 | -20.9              | 23.1                       | -25.2              | 27.0                 |  |
| Lee and Lee                   | -47.8              | 49.5                 | -43.6              | 46.6                 | -30.0              | 32.3                       | -34.9              | 36.4                 |  |
| Qu and Mudawar                | -26.4              | 29.3                 | -23.9              | 28.5                 | -9.3               | 16.1                       | -11.4              | 19.6                 |  |
| Eq. (14) with <i>b</i> = 1.38 | -15.9              | 20.7                 | -17.0              | 21.6                 | -12.6              | 15.0                       | 9.2                | 13.7                 |  |
| Separated flow model          | -17.6              | 25.0                 | -13.1              | 15.7                 | 2.7                | 30.8                       | -14.3              | 18.6                 |  |
| (b) With contraction          |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Homogeneous flow type         |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Owens                         | 28.2               | 40.5                 | 54.9               | 114.9                | 77.8               | 99.9                       | 75.5               | 110.9                |  |
| McAdams                       | 18.0               | 28.0                 | 31.5               | 59.1                 | 41.0               | 46.6                       | 53.1               | 71.2                 |  |
| Cicchitti                     | 28.0               | 40.3                 | 54.4               | 113.6                | 77.4               | 99.3                       | 75.1               | 110.1                |  |
| Dukler et al.                 | -26.7              | 33.9                 | -22.8              | 27.7                 | -2.2               | 10.4                       | -3.6               | 8.3                  |  |
| Beattie and Whalley           | 39.2               | 49.0                 | 46.1               | 55.2                 | 87.4               | 95.1                       | 83.4               | 93.4                 |  |
| Lin et al.                    | 27.2               | 39.2                 | 50.6               | 101.6                | 72.7               | 90.9                       | 72.6               | 104.5                |  |
| Lockhart–Martinelli type      |                    |                      |                    |                      |                    |                            |                    |                      |  |
| Chisholm and Laird            | 9.4                | 21.6                 | 15.8               | 23.9                 | 26.0               | 27.2                       | 41.2               | 45.4                 |  |
| Mishima and Hibiki            | -17.2              | 25.8                 | -10.4              | 17.0                 | 5.1                | 8.4                        | 10.8               | 12.5                 |  |
| Lee and Lee                   | -29.6              | 36.0                 | -22.7              | 27.6                 | -4.8               | 11.0                       | -3.6               | 8.3                  |  |
| Qu and Mudawar                | 3.2                | 25.8                 | 2.4                | 17.8                 | 13.3               | 18.4                       | 22.1               | 24.4                 |  |
| Eq. (14) with <i>b</i> = 0.55 | -7.6               | 22.8                 | -0.02              | 15.4                 | 8.2                | 11.8                       | 17.4               | 18.8                 |  |
| Separated flow model          | 11.1               | 25.2                 | 32.6               | 97.1                 | 32.8               | 36.5                       | 44.5               | 70.4                 |  |

ity model (1964) give the best predictions, irrespective of working liquids. Besides L–M model with Eq. (14), L–M model with Qu and Mudawar's C model (2003) predicts reasonably well both flow data without and with the contraction, irrespective of working liquids. Figs. 13 and 14 show a graphical presentation of the prediction results by L–M method with Eq. (14) and Qu and Mudawar's C model (2003), respectively. The RMS error is within 30% for both flows with and without the contraction, irrespective of the test liquids, and Eq. (14) has better prediction as a result of accounting the liquid properties. The reason of the better prediction by Qu and Mudawar's C model is probably that the model is modified version

of Mishima and Hibiki's one to incorporate the effects of mass flux based on the data in microchannel heat sink containing 21 parallel  $271 \times 713 \ \mu m$  microchannels.

# 4.2. Void fraction prediction

Ten traditional correlations developed for both macro- and mini/micro-channels are examined to validate their suitability for use with the present void fraction data. The correlations tested are: Armand type correlations (Armand, 1946; Chisholm, 1973; Spedding and Chen, 1986), correlations of homogeneous flow etc.



Fig. 13. Comparison of two-phase frictional pressure gradient between experiments and calculations by L-M model with a newly developed C correlation, Eq. (14).



Fig. 14. Comparison of two-phase frictional pressure gradient between experiments and calculation by L-M model with Qu and Mudawar's C model (2003).

# Table 6 Mean and RMS errors of various correlations for predicting void fraction.

|                                     | Distilled water |                  | Ethanol 4.8 wt% |                  | Ethanol 49 wt% |                  | Ethanol 100 wt% |                  |
|-------------------------------------|-----------------|------------------|-----------------|------------------|----------------|------------------|-----------------|------------------|
|                                     | E <sub>M</sub>  | E <sub>RMS</sub> | E <sub>M</sub>  | E <sub>RMS</sub> | E <sub>M</sub> | E <sub>RMS</sub> | E <sub>M</sub>  | E <sub>RMS</sub> |
| (a) Without contraction             |                 |                  |                 |                  |                |                  |                 |                  |
| Armand type correlation             |                 |                  |                 |                  |                |                  |                 |                  |
| Armand correlation                  | -0.031          | 0.044            | -0.041          | 0.050            | 0.053          | 0.087            | 0.033           | 0.053            |
| Chisholm correlation                | -0.028          | 0.042            | -0.036          | 0.051            | 0.055          | 0.086            | 0.041           | 0.054            |
| Speeding and Chen                   | -0.031          | 0.044            | -0.041          | 0.050            | 0.054          | 0.087            | 0.034           | 0.053            |
| Butterworth type correlation        |                 |                  |                 |                  |                |                  |                 |                  |
| Homogeneous flow model              | 0.030           | 0.055            | 0.022           | 0.035            | 0.122          | 0.151            | 0.088           | 0.110            |
| Zivi model                          | -0.277          | 0.298            | 0.279           | 0.306            | -0.191         | 0.220            | -0.174          | 0.191            |
| Turner and Wallis model             | -0.284          | 0.310            | -0.285          | 0.317            | -0.208         | 0.242            | -0.177          | 0.197            |
| Lockhart and Martinelli correlation | -0.113          | 0.131            | -0.119          | 0.142            | -0.031         | 0.083            | -0.019          | 0.042            |
| Thom correlation                    | -0.203          | 0.217            | -0.215          | 0.233            | -0.134         | 0.156            | -0.121          | 0.133            |
| Baroczy correlation                 | -0.154          | 0.169            | -0.163          | 0.183            | -0.086         | 0.116            | -0.070          | 0.083            |
| Kawahara et al. correlation         | 0.050           | 0.057            | 0.008           | 0.035            | 0.004          | 0.063            | 0.090           | 0.098            |
| (b) With contraction                |                 |                  |                 |                  |                |                  |                 |                  |
| Armand type correlation             |                 |                  |                 |                  |                |                  |                 |                  |
| Armand                              | -0.021          | 0.136            | 0.018           | 0.060            | 0.070          | 0.084            | 0.032           | 0.064            |
| Chisholm                            | -0.021          | 0.136            | 0.022           | 0.066            | 0.073          | 0.083            | 0.036           | 0.069            |
| Speeding and Chen                   | -0.021          | 0.136            | 0.019           | 0.060            | 0.070          | 0.084            | 0.032           | 0.064            |
| Butterworth type correlation        |                 |                  |                 |                  |                |                  |                 |                  |
| Homogeneous flow model              | 0.053           | 0.148            | 0.085           | 0.104            | 0.135          | 0.157            | 0.096           | 0.117            |
| Zivi model                          | -0.290          | 0.334            | -0.212          | 0.249            | -0.164         | 0.183            | -0.195          | 0.228            |
| Turner and Wallis model             | -0.303          | 0.349            | -0.226          | 0.273            | -0.180         | 0.207            | -0.206          | 0.248            |
| Lockhart and Martinelli correlation | -0.118          | 0.187            | -0.062          | 0.112            | -0.014         | 0.043            | -0.037          | 0.087            |
| Thom correlation                    | -0.209          | 0.255            | -0.144          | 0.167            | -0.116         | 0.128            | -0.136          | 0.158            |
| Baroczy correlation                 | -0.160          | 0.217            | -0.102          | 0.136            | -0.068         | 0.082            | -0.088          | 0.120            |
| Kawahara et al.                     | 0.036           | 0.138            | 0.044           | 0.079            | -0.021         | 0.037            | 0.042           | 0.070            |

seen in Butterworth's paper (1975) and Kawahara et al.'s correlation (2005b). Table 6 lists the mean error,  $E_M$ , and the RMS error,  $E_{RMS}$ , in each correlation, defined as

$$E_{M} = \frac{1}{N} \sum_{i=1}^{N} (\alpha_{Cal,i} - \alpha_{Exp,i}),$$
(17)

$$E_{RMS} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\alpha_{Cal,i} - \alpha_{Exp,i})^2}.$$
 (18)

Chisholm (1973), Armand (1946), Spedding and Chen (1986) and Kawahara et al. (2005b) correlations give better results. In more detail, Chisholm (1973), Armand (1946) and Spedding and Chen (1986) correlations tend to over-predict the data for flows with the contraction, while Kawahara et al.'s one over-predict the data for flows without the contraction. Fig. 15 shows a graphical representation of the prediction results by Kawahara et al.'s correlation (2005b). The reason of the better prediction by Kawahara et al.'s correlation is probably that the correlation was developed by using data for two-phase flows in 50–250  $\mu$ m microchannels.

Two-fluid model (Ishii, 1975; Ishii and Mishima, 1984) is an upto-date prediction method, and is used in various engineering fields. So, we tried to test the two-fluid model against the present void fraction data. In the calculation of a steady state, adiabatic flow by the two-fluid model, the following gas and liquid momentum equations were simultaneously solved:

$$\frac{d}{dZ}\left(\rho_{G}\alpha u_{G}^{2}\right)+F_{WG}+F_{I}+\alpha\frac{dP_{G}}{dZ}=0,$$
(19)

$$\frac{d}{dZ}\left(\rho_L(1-\alpha)u_L^2\right) + F_{WL} - F_I + (1-\alpha)\frac{dP_L}{dZ} = 0.$$
(20)



Fig. 15. Comparison of void fraction between experiment and calculation by Kawahara et al. correlation (2005b).



Fig. 16. Comparison of void fraction between experiment and calculation by the two-fluid model.

# Table 7Mean and RMS errors of two-fluid model for predicting void fraction.

|   | Distilled water  |                | Ethanol 4.8 wt% |                | Ethanol 49 wt% | <u></u> _      | Ethanol 100 wt% |                |
|---|------------------|----------------|-----------------|----------------|----------------|----------------|-----------------|----------------|
|   | Em               | Erms           | Em              | Erms           | Em             | Erms           | Em              | Erms           |
| Without contraction<br>With contraction | -0.001<br>-0.008 | 0.030<br>0.028 | -0.026<br>0.004 | 0.037<br>0.065 | 0.020<br>0.016 | 0.068<br>0.032 | 0.001<br>-0.028 | 0.029<br>0.074 |

Here,  $u_k$  is the mean velocity of k-phase (k = G for gas, k = L for liquid),  $F_{Wk}$  and  $F_l$  the wall friction force of k-phase and the gas-liquid interfacial friction force per unit volume. In the present calculation, the wall friction force for the gas phase,  $F_{WG}$ , was taken to be zero by considering the present experimental range. Thus, correlations of  $F_l$  and  $F_{WL}$ , being the same as two-phase frictional pressure drop, were required as the constitutive equations.

In our previous study on the prediction of  $F_l$  in a triangle tight lattice subchannel, having about 3 mm in hydraulic diameter (Kawahara et al., 2008), the following Tomiyama et al.'s correlation (1993) showed the best results.

$$F_{I} = \frac{1}{8} a_{INT} C_{D} \rho_{L} (u_{G} - u_{L}) |u_{G} - u_{L}|, \qquad (21)$$

$$a_{INT}C_{D} = \frac{8\{\alpha(1-\alpha)(\rho_{L}-\rho_{G})g - (1-\alpha)F_{WG} + \alpha F_{WL}\}}{\rho_{L}u_{R}|u_{R}|},$$
(22)

$$u_{R} = \frac{V_{Gj} + (C_{0} - 1)u_{L}}{1 - C_{0}\alpha},$$
(23)

So, Tomiyama et al.'s  $F_I$  correlation was used in this study. For  $C_0$ , we used Eq. (7) developed in this study. For  $F_{WL}$ , on the other side, we used the L–M model with the developed *C* correlation, Eq. (14), being the best in Section 4.1.

Fig. 16 shows a comparison of void fraction between experiment and calculation by the above two-fluid model. In the calculation of  $F_I$  by Eqs. (21)–(23),  $V_{G_I}$  and  $F_{WG}$  are taken as zero. Table 7 shows the mean and the RMS errors of the two-fluid model calculation. The calculation could predict the data within RMS error of 0.08 for both flows without and with the contraction, irrespective of working liquids.

# 5. Conclusions

The characteristics of adiabatic two-phase flows in a horizontal circular microchannel have been investigated experimentally and analytically. In the experiments, in order to determine the effects of fluid properties on the flow characteristics, distilled water and aqueous solutions of ethanol having three different mass concentrations were used as the working liquids. One of the four liquids and nitrogen gas were injected through a T-junction type mixer to the test microchannel made of fused silica. To know the effects of flow contraction at the channel inlet, two mixers of different inner diameters of  $D_M$  = 250 µm and 500 µm were used at a fixed microchannel diameter of D = 250 µm. In the analysis, the two-phase frictional pressure drop and the void fraction correlations from literatures were tested against the present data. Furthermore, on the void fraction, an analytical code based on a steady state, adiabatic one-dimensional two-fluid model was also tested. The main findings are as follows.

- (1) The bubble velocity,  $u_G$ , depends on both the liquid properties and the flow contraction, i.e., the distribution parameter,  $C_0$ , in the drift flux model increased with increasing of the liquid viscosity and/or decreasing of surface tension, and were higher for the flows with the contraction. The  $C_0$  data were correlated well with three dimensionless numbers, i.e., Bond number, *Bo*, liquid Reynolds number, *Re*<sub>L</sub>, and gas Weber number, *We*<sub>G</sub>.
- (2) For the flows with the contraction, bubbles were elongated and flowed faster, thus the void fraction became lower.
- (3) The void fraction decreased with increasing of the liquid viscosity and/or decreasing of the surface tension because the bubbles flowed faster with increasing of the liquid viscosity and/or decreasing of the surface tension.
- (4) Two-phase friction multiplier in Lockhart and Martinelli (L-M) method (1949) was lower for the flows with the contraction than that without the contraction.
- (5) L–M method with a newly developed C correlation accounting liquid properties and Qu and Mudawar's one (2003) gave the best prediction for the present data for both flows without and with the flow contraction, irrespective of working liquids.
- (6) Armand type correlation (Armand, 1946; Chisholm, 1973; Spedding and Chen, 1986) and Kawahara et al.'s correlation (2005b) could predict well the present void fraction data.
- (7) On the void fraction, two-fluid model code also could predict well the data when Tomiyama et al.'s  $F_I$  correlation (1993) with the newly developed correlations of  $C_0$  and  $F_W$  were incorporated.

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